التحليل الاحتمالي لنظام مكون من وحدتين احدهما موصلة والأخرى احتياط (من النوع الذي قد يعطل وهو في حالة الاحتياط)غير متماثلتين موصلتين بمحول

د . انتصار عبدالله الصويعي السائح ـ المعهد العالي للعلوم والتقنية ـ العزيزية

ملخص البحث :

تتناول هذه الورقة نظام الاعتماد المكون من وحدتين غير متماثلتين حيث تكون الوحدة الاحتياطية محملة (ممكن تتعطل الوحدة الاحتياطية أثناء تشغيل الوحدة العاملة في النظام) بافتراض إن الوحدة العاملة لها مستويان للقدرة (كفاءة تامة أو عطل كلي) ويتم الإحلال بين الوحدتين الاحتياطية والوحدة العاملة في حالة تعطل الوحدة العاملة عن طريق المحول بافتراض إن المحو ل له مستويان للقدرة (كفاءة تامة أو عطل كلي) وفى حالة تعطل إي من الوحدة العاملة ,الوحدة الاحتياطية و المحول يتم تصليح العطل ، وبافتراض أن أزمنة التعطل والتصليح متغيرات عشوائية تتبع توزيعات مختلفة وعامة ، وأمكن الحصول على دالة الاعتماد ومتوسط عمر النظام ومعامل النفاعية في حالة الاستقرار ، ونتائج هذه الورقة مدعمة عدديا وبيانيا باستخدام برامج الحاسب الآلي. (باستخدام برنامج (wolfram mathematica)

PROBABILISTIC ANALYSIS OF TWO-DISSIMILAR UNIT WARM STANDBY REDUNDANT SYSTEM WITH IMPERFECT SWITCH

Entesar Al- Esayeh

Higher Institute Of Sciences And Technology-Azizia

intesaralsayah2@gmail.com

Abstract: Problem statement: This study presents the statistical analysis of a two-dissimilar unit warm standby redundant system with two modes (normal and total failure) and imperfect switching device has been studied. Failure, repair of units and repair of a switch time distributions are stochastically independent random variables each having an arbitrary distribution. The system is analyzed by the semi-Markov process technique. Some reliability measures of interest to system designers as well as operations managers have been obtained. Explicit expressions for the Laplace-Stieltjes transforms of the distribution function of the first passage time, mean time to system failure, steady state availability and the busy period analysis of the system are obtained. Certain important results have been derived as particular cases.

INTRODUCTION AND DESCRIPTION OF THE SYSTEM

Many authors [1, 2, and 5] have studied the two-unit warm standby system operating under different model formulations by using the theory of semi-Markov process, regenerative process and Markov renewal process. They obtained the mean time to system failure, the pointwise availability and the steady state availability of the system using the theory of regenerative process.

The purpose of the present chapter is to investigate a two-dissimilar-unit warm standby system where each unit works in two different modes normal and total failure. The failure, the repair of units and the repair of a switch time distributions are assumed to different arbitrary distributed, the probability that the switch works at the time of reed is $q = (1 - p)$. Repair of a total failure unit, failed from operative or standby state continues when the other unit enters the total failure mode, when an operative unit fails, switch is used to disconnect the failed unit and connect the standby unit if it is operative. A single repair facility is available, priority for repair being given to transfer switch, after repair of a unit or the switch works like a new one, repair time distribution of a unit failed from the standby state is different from that of the unit failed from operative state. Switch failure occurs in a non-regenerative state, for the first time. Using the semi-Markov process technique and the results of the regenerative process, various measures of the system effectives as mean time to system failure, pointwise availability, steady state availability and busy period analysis are found out. The results by [6] are derived from the present results as a special case.

The following systems characteristics are studied:-

i. Mean time to system failure.

ii. Pointwise availability of the system at time t and steady state availability.

iii. Busy period analysis of the system.

iv. Profit analysis of the system.

The following assumptions are adopted for the system:-

1. The system consists of two-dissimilar units in warm standby configuration, so that a unit can fail during its standby state,
2. Each unit has two modes normal and total failure,
3. Failure of units, repair of units and repair of a switch times are stochastically independent random variables each having an arbitrary distribution,
4. The probability that the switch works at the time of need is $p=(1-q)$,
5. A single repair facility is available. Priority for repair being given to transfer switch. After repair of a unit or the switch works like a new one,
6. Repair of a total failure unit, failed from operative or standby state continues when the other unit enters the total failure mode from the normal mode,
7. When an operative unit fails, switch is used to disconnect the failed unit and connect the standby unit(if it is operative),
8. Switch failure occurs in a non-regenerative state, for the first time,
9. Repair time distribution of a unit failed from the standby state is different from that of the unit failed from operative state.

NOTATIONS AND STATES OF THE SYSTEM

E_0	State of the system at time point $t=0$,
E	set of regenerative states; $\{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$,
$\bar{E}$	set of non-regenerative states; $\{S_8, S_9, S_{10}, S_{11}\}$,
$f_i(t)$, $F_i(t)$	pdf and cdf of failure time of the i^{th} operative unit from normal mode to total failure mode; $i=1,2$,
$l_i(t)$, $L_i(t)$	pdf and cdf of failure time of the i^{th} standby unit from normal mode to total failure mode; $i=1,2$,
$g_i(t)$, $G_i(t)$	pdf and cdf of time to repair for the i^{th} unit failed while operative; $i=1,2$,
$h_i(t)$, $H_i(t)$	pdf and cdf of time to repair for a unit failed while in standby; $i=1,2$,
$j_i(t)$, $J_i(t)$	pdf and cdf of time to repair for the switch,
$q_{ij}(t)$, $Q_{ij}(t)$	pdf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j

	without visiting any other regenerative state in (0,t]; $i, j \in E$,
$q_{ij}^{(k)}(t)$, $Q_{ij}^{(k)}(t)$	pdf and cdf of first passage time from regenerative state i to a regenerative state j or to failed state j visiting state k only once in (0,t] ; $i, j \in E; k \in \bar{E}$,
$q_{ij}^{(k,h)}(t)$, $Q_{ij}^{(k,h)}(t)$	pdf and cdf of first passage time from regenerative state i to a regenerative state j or failed state j visiting states k and h in (0,t] respectively; $i, j \in E; h,k \in \bar{E}$,
p_{ij}	one step transition probability from state i to state j; $i, j \in E$,
$p_{ij}^{(k)}$	probability that the system in state i goes to state j passing through state k; $i, j \in E; k \in \bar{E}$
$p_{ij}^{(k,h)}$	probability that the system in state i goes to state j passing through states k,h; $i, j \in E; h,k \in \bar{E}$
$\pi_i(t)$	cdf of first passage time from regenerative state i

to a failed state,

$A_i(t)$ probability that the system is in up state at instant t given that the system started from regenerative state i at time $t=0$,

$M_i(t)$ probability that the system, having started from state i is up at time t without making any transition into any other regenerative state,

$B_i(t)$ probability that the repairman is busy at time t given that the system entered regenerative state i at time $t=0$,

$V_i(t)$ expected number of visits by the repairman given that the system started from

μ_{ij} contribution mean sojourn time in state I when transition is to state j is $-\tilde{Q}_{ij}(0)=q_{ij}^{*}(0)$,

$\mu_i(t)$ Mean sojourn time in state i

$$\mu_i=\sum_j\left[\mu_{ij}+\sum_k\mu_{ij}^{(k)}\right],$$

~ symbol for Laplace-Stieltjes transform, e.g.

$$\tilde{F}(s) = \int e^{-st} dF(t),$$

* symbol for Laplace transform, e.g.

$$f^*(s) = \int e^{-st} f(t) dt,$$

(s) symbol for Stieltjes convolution, e.g.

$$A(t)(s)B(t) = \int_0^t B(t-u)dA(u),$$

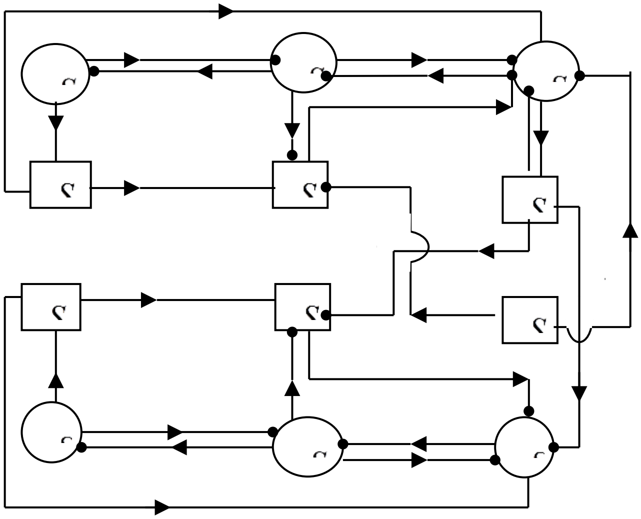
© symbol for ordinary convolution, e.g.

$$a(t) © b(t) = \int_0^t a(u)b(t-u)du,$$

For simplicity, whenever integration limits are they $(0,\infty)$, are not written.

STOCHASTIC BEHAVIOR OF THE SYSTEM

States and possible transitions between them are shown in Figure (1).



○Up state □ :Down state • : Regeneration point.

Figure 1: State transition diagram

The system can take one of the following states:-

1. $S_0(N_{O1}, N_{\bar{S}2})$: The first unit is operative in normal mode and the second unit is standby in normal mode.

2. $S_1(N_{\bar{S}1}, N_{O2})$: The first unit is standby in normal mode and the second unit is operative in normal mode.

3. $S_2(F_{Or1}, N_{O2})$: The first unit is operative in total failure mode and under repair and the second unit is operative in normal mode.

4. $S_3(N_{O1}, F_{Or2})$: The first unit is operative in normal mode and the second unit is operative in total failure mode and under repair.

5. $S_4(N_{O1}, F_{Sr2})$: The first unit is operative in normal mode and the second unit is standby in total failure mode and under repair.

6. $S_5(F_{Sr1}, N_{O2})$: The first unit is standby in total failure mode and under repair and under repair and the second unit is operative in normal mode.

7. $S_6(F_{OW1}, N_{\bar{S}2}, S_r)$: The first unit is operative in total failure mode and waiting for repair, the second unit is standby in normal mode and switching device is under repair.

8. $S_7(N_{\bar{S}1}, F_{OW2}, S_r)$: The first unit is standby in normal mode, second unit is operative in total failure mode and waiting for repair and switching device is under repair.

9. $S_8(F_{OR1}, F_{OW2})$: The first unit is operative in total failure mode, with repair continued from earlier state and the second unit is operative in total failure mode and waiting for repair.

10. $S_9(F_{OW1}, F_{OR2})$: The first unit is operative in total failure mode and waiting for repair and the second unit is operative in total failure mode, with repair continued from earlier state.

11. $S_{10}(F_{OW1}, F_{SR2})$: The first unit is operative in total failure mode and waiting for repair and the second unit is standby in total failure mode, with repair continued from earlier state.

12. $S_{11}(F_{SR1}, N_{OW2})$: The first unit is operative in total failure mode, with repair continued from earlier state and the second unit is operative in total failure mode and waiting for repair.

Where,

N_{Oi}	the i^{th} unit is operative in normal mode; $i = 1, 2$,
$N_{\bar{S}i}$	the i^{th} unit is standby in normal mode; $i = 1, 2$,
F_{Ori}	the i^{th} unit is operative in total failure mode and under repair; $i = 1, 2$,
$F_{\bar{S}ri}$	the i^{th} unit is standby in total failure mode and under repair; $i = 1, 2$,
F_{ORi}	the i^{th} unit is operative in total failure mode , with repair continued from earlier state; $i = 1, 2$,
$F_{\bar{S}Ri}$	the i^{th} unit is standby in total failure mode , with repair continued from earlier state; $i = 1, 2$,
F_{OWi}	the i^{th} unit is operative in total failure mode and waiting for repair; $i = 1, 2$,
S_r	switching device is under repair.

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

It can be observed that the time points of entry into $S_i \in E$ are regenerative points so these states are regenerative. Let $T_0(\equiv 0), T_1, T_2, ..$denote the time points at which the system enters any state $S_i \in E$ and X_n denotes the state visited at the time point i.e.T_{n+1}, just after the transition at T_{n+1}, then$\{X_n, T_n\}$is a Makov-renewal process with state space E and $Q_{ij} = p[X_{n+1} = j, T_{n+1} - T_n < t / X_n = i]$is a semi-Markov kernel over E. The stochastic matrix of the embedded Markov chain is $P = (P_{ij}) = (Q_{ij}(\infty)) = Q(\infty)$and the nonzero elements P_{ij} are

$P_{02} = p\int \bar{L}_2(t)\, dF_1(t)$, $P_{04} = \int \bar{F}_1(t)\, dL_2(t)$,

$P_{06} = q\int \bar{L}_2(t)\, dF_1(t)$, $P_{13} = p\int \bar{L}_1(t)\, dF_2(t)$,

$P_{15} = \int \bar{F}_2(t)\, dL_1(t)$, $P_{17} = q\int \bar{L}_1(t)\, dF_2(t)$,

$P_{20} = \int \bar{F}_2(t)\, dG_1(t)$, $P_{28} = p\int \bar{G}_1(t)\, dF_2(t)$,

$P_{23}^{(8)} = p\int F_2(t)\, dG_1(t)$, $P_{27}^{(8)} = q\int F_2(t)\, dG_1(t)$,

$P_{31} = \int \bar{F}_1(t)\, dG_2(t)$, $P_{39} = p\int \bar{G}_2(t)\, dF_1(t)$,

$P_{32}^{(9)} = p\int F_1(t)\, dG_2(t)$, $P_{36}^{(9)} = q\int F_1(t)\, dG_2(t)$,

$P_{40} = \int \bar{F}_1(t)\, dH_2(t)$, $P_{4,10} = \int \bar{H}_2(t)\, dF_1(t)$,

$P_{42}^{(10)} = p\int F_1(t)\, dH_2(t)$, $P_{46}^{(10)} = q\int F_1(t)\, dH_2(t)$,

$P_{51} = \int \bar{F}_2(t)\, dH_1(t)$, $P_{5,11} = \int \bar{H}_1(t)\, dF_2(t)$,

$P_{53}^{(11)} = p\int F_2(t)\, dH_1(t)$, $P_{57}^{(11)} = q\int F_2(t)\, dH_1(t)$,

$P_{62} = P_{73} = 1$.

The mean sojourn times μ_i in state S_i are

$\mu_0 = \int \bar{F}_1(t)\ \bar{L}_2(t)dt$, $\mu_1 = \int \bar{F}_2(t)\ \bar{L}_1(t)dt$,

$\mu_2 = \int \bar{G}_1(t)\ \bar{F}_2(t)dt$, $\mu_3 = \int \bar{G}_2(t)\ \bar{F}_1(t)dt$,

$\mu_4 = \int \bar{F}_1(t)\ \bar{H}_2(t)dt$, $\mu_5 = \int \bar{F}_2(t)\ \bar{H}_1(t)dt$,

$\mu_6 = \mu_7 = \int \bar{J}(t)dt$, $\mu_8 = \int \bar{G}_1(t)\ dt$,

$\mu_9 = \int \bar{G}_2(t)dt$, $\mu_{10} = \int \bar{H}_2(t)\ dt$,

$\mu_{11} = \int \bar{H}_1(t)dt$.

MEANTIME TO SYSTEM FAILURE

Time to system failure can be regarded as the first passage to the failed states $S_6, S_7, S_8, S_9, S_{10}, S_{11}$ which are considered as absorbing. By probabilistic arguments, the following recursive relations for $\pi_i(t)$ are obtained.

$$\pi_0 = Q_{02}(t)\ (s)\ \pi_2 + Q_{04}(t)\ (s)\ \pi_4 + Q_{06}(t),$$

$$\pi_1 = Q_{13}(t)\ (s)\ \pi_3 + Q_{15}(t)\ (s)\ \pi_5 + Q_{17}(t),$$

$$\pi_2 = Q_{20}(t)\ (s)\ \pi_0 + Q_{28}(t),$$

$$\pi_3 = Q_{31}(t)\ (s)\ \pi_1 + Q_{39}(t),$$

$$\pi_4 = Q_{40}(t)\ (s)\ \pi_0 + Q_{4,10}(t),$$

$$\pi_5 = Q_{51}(t)\ (s)\ \pi_1 + Q_{5,11}(t).$$

(1)

Taking Laplace-Stieltjes transforms of equations (1) and solving for $\tilde{\pi}_0(s)$, dropping the argument “s” for brevity, we get

$\tilde{\pi}_0(s) = N_0(s)/D_0(s)$,

(2)

where,

$$N_0(s) = \left(1 - \tilde{Q}_{13}\tilde{Q}_{31} - \tilde{Q}_{15}\tilde{Q}_{51}\right)\left(\tilde{Q}_{06} + \tilde{Q}_{02}\tilde{Q}_{28} + \tilde{Q}_{04}\tilde{Q}_{4,10}\right)$$

and

$$D_0(s) = \left(1 - \tilde{Q}_{13}\tilde{Q}_{31} - \tilde{Q}_{15}\tilde{Q}_{51}\right)\left(1 - \tilde{Q}_{02}\tilde{Q}_{20} - \tilde{Q}_{04}\tilde{Q}_{40}\right).$$

The mean time to system failure with starting state S_0 **is given by**

$$MTSF = N_0 / D_0, \tag{3}$$

where,

$$N_0 = \left(\mu_0 + P_{02}\mu_2 + P_{04}\mu_4\right)\left[1 - P_{13}P_{31} - P_{15}P_{51}\right]$$
$$+\left(P_{31}\mu_1 + P_{13}\mu_3 + P_{15}\mu_5\right)\left[1 - P_{06} - P_{02}\left(P_{20} + P_{28}\right) - P_{04}\left(P_{40} + P_{4,10}\right)\right]$$

and

$$D_0 = \left(1 - P_{13}P_{31} - P_{15}P_{51}\right)\left(1 - P_{02}P_{20} - P_{04}P_{40}\right).$$

AVAILABILITY ANALIYSIS

The following probabilistic argument

$M_0(t) = \bar{F}_1(t)\bar{L}_2(t)$, $M_1(t) = \bar{F}_2(t)\bar{L}_1(t)$,

$M_2(t) = \bar{G}_1(t)\bar{F}_2(t)$, $M_3(t) = \bar{G}_2(t)\bar{F}_1(t)$,

$M_4(t)=\bar{F}_1(t)\bar{H}_2(t)$, $M_6(t)=\bar{F}_2(t)\bar{H}_1(t)$.

Can be used in the theory of regenerative process, in order to find the pointwise availabilities $A_i(t)$ are seen to satisfy the following recursive relations:-

$$A_0(t)=M_0(t)+q_{02}(t)© A_2(t)+q_{04}(t)©A_4(t)+q_{06}(t)©A_6(t) ,$$

$$A_1(t)=M_1(t)+q_{13}(t)© A_3(t)+q_{15}(t)©A_5(t)+q_{17}(t)©A_7(t),$$

$$A_2(t)=M_2(t)+q_{20}(t)© A_0(t)+q_{23}^{(8)}(t)©A_3(t)+q_{27}^{(8)}(t)©A_7(t),$$

$$A_3(t)=M_3(t)+q_{31}(t)© A_1(t)+q_{32}^{(9)}(t)©A_2(t)+q_{36}^{(9)}(t)©A_6(t),$$

$$A_4(t)=M_4(t)+q_{40}(t)© A_0(t)+q_{42}^{(10)}(t)©A_2(t)+q_{46}^{(10)}(t)©A_6(t),$$

$$A_5(t)=M_5(t)+q_{51}(t)© A_1(t)+q_{53}^{(11)}(t)©A_3(t)+q_{57}^{(11)}(t)©A_7(t),$$

$$A_6(t)=q_{62}(t)© A_2(t),$$

$$A_7(t)=q_{73}(t)© A_3(t). \tag{4}$$

After using Laplace-stieltjes transforms for equations (4) and solving for $A_0^*(s)$ it can be easily seen that. The steady state availability of the system is

$$A_0=\lim_{s\to 0} sA_0(s)=\frac{N_2(0)}{D_2'(0)}, \tag{5}$$

where,

$$N_2(0)=(\mu_0+\mu_4P_{04})\left[(1-P_{31}P_{13}-P_{15}P_{51}-P_{31}(P_{17}+P_{15}\left(P_{53}^{11}+P_{57}^{11}\right)\right)$$
$$-(1-P_{15}P_{51})\left(P_{32}^{9}+P_{36}^{9}\right)\left(P_{23}^{8}+P_{27}^{8}\right)\Big]\,(\mu_1+P_{15}\mu_5)[P_{31}\left(P_{23}^{8}+P_{27}^{8}\right)$$

$$\left(P_{02}+P_{06}+P_{04}\left(P_{42}^{10}+P_{46}^{10}\right)\right)\Big]+\mu_2\Big[\big(1-P_{31}P_{13}-P_{15}P_{51}-$$

$$P_{31}(P_{17}+P_{15}\left(P_{53}^{11}+P_{57}^{11}\right)\big)\left(P_{02}+P_{06}+P_{04}\left(P_{42}^{10}+P_{46}^{10}\right)\right)\Big]+\mu_3\Big[\left(P_{23}^{8}+P_{27}^{8}\right)$$

$$\left(1-P_{15}P_{51}\right)\left(P_{02}+P_{06}+P_{04}\left(P_{42}^{10}+P_{46}^{10}\right)\right)\Big],$$

$$D_1'(0)=\Big[P_{20}\mu_0+\mu_2\left(P_{02}+P_{06}+P_{04}\left(P_{42}^{10}+P_{46}^{10}\right)\right)\Big]\big(1-P_{13}P_{31}-P_{15}P_{51}-$$

$$P_{31}\left(P_{17}+P_{15}\left(P_{53}^{11}+P_{57}^{11}\right)\right)\big)\Big)+\left[\mu_1P_{31}+\mu_3\right]\big(1-P_{20}\left(P_{02}-P_{06}\right)-P_{04}\big(P_{40}+P_{20}$$

$$\left(P_{42}^{10}+P_{46}^{10}\right)\big)\big)+\mu_4\Big[P_{04}\Big\{\big(1-P_{13}P_{31}-P_{15}P_{51}-P_{31}$$

$$\left(P_{17}+P_{15}\left(P_{53}^{11}+P_{57}^{11}\right)\right)\big)-\left(1-P_{15}P_{51}\right)\left(P_{23}^{8}+P_{27}^{8}\right)\left(P_{32}^{9}+P_{36}^{9}\right)\Big\}\Big]$$

$$+\mu_5\Big[P_{15}\Big(\big(1-P_{20}\left(P_{02}-P_{06}\right)-P_{04}\left(P_{40}+P_{20}\left(P_{42}^{10}+P_{46}^{10}\right)\right)\big)$$

$$-\left(1-P_{04}P_{40}\right)\left(P_{23}^{8}+P_{27}^{8}\right)\left(P_{32}^{9}+P_{36}^{9}\right)\Big\}\Big]+\mu_6\Big[P_{20}\left(P_{06}+P_{04}P_{46}^{10}\right)$$

$$\left(1-P_{13}P_{31}-P_{15}P_{51}-P_{31}\left(P_{17}+P_{15}\left(P_{53}^{11}+P_{57}^{11}\right)\right)\right)+P_{36}^{9}\left(1-P_{04}P_{40}\right)$$

$$\left(1-P_{15}P_{51}\right)\left(P_{23}^{8}+P_{27}^{8}\right)\Big]+\mu_7\Big[P_{31}\left(P_{17}+P_{15}P_{57}^{11}\right)$$

$$\Big(\big(1-P_{20}\left(P_{02}-P_{06}\right)-P_{04}\left(P_{40}+P_{20}\left(P_{42}^{10}+P_{46}^{10}\right)\right)\big)\Big)$$

$$+P_{27}^{8}\left(P_{32}^{9}+P_{36}^{9}\right)\left(1-P_{04}P_{40}\right)\left(1-P_{15}P_{51}\right)\Big].$$

BUSY PERIOD ANALYSIS

By probabilistic arguments we have:

$V_2(t)=\bar{G}_1(t)\bar{F}_2(t)$, $\quad$ $V_3(t)=\bar{G}_2(t)\bar{F}_1(t)$,

$V_4(t)=\bar{F}_1(t)\bar{H}_2(t)$, $\quad$ $V_5(t)=\bar{F}_2(t)\bar{H}_1(t)$,

$V_6(t)=V_7(t)=\bar{J}(t)$.

For busy period analysis, let $B_i(t)$ be the probability that the operative unit is under repair at time t given that the system entered regenerative state S_i at $t=0$. From previous probabilistic argument, the following relations for $B_i(t)$ are given.

$$B_0(t) = q_{02}(t) © B_2(t) + q_{04}(t) © B_4(t) + q_{06}(t) © B_6(t) \ ,$$

$$B_{01}(t) = q_{13}(t) © B_3(t) + q_{15}(t) © B_5(t) + q_{17}(t) © B_7(t),$$

$$B_2(t) = V_2(t) + q_{20}(t) © B_0(t) + q_{23}^{(8)}(t) © B_3(t) + q_{27}^{(8)}(t) © B_7(t),$$

$$B_3(t) = V_3(t) + q_{31}(t) © B_1(t) + q_{32}^{(9)}(t) © B_2(t) + q_{36}^{(9)}(t) © B_6(t),$$

$$B_4(t) = V_4(t) + q_{40}(t) © B_0(t) + q_{42}^{(10)}(t) © B_2(t) + q_{46}^{(10)}(t) © B_6(t),$$

$$B_5(t) = V_5(t) + q_{51}(t) © B_1(t) + q_{53}^{(11)}(t) © B_3(t) + q_{57}^{(11)}(t) © B_7(t),$$

$$B_6(t) = V_6(t) + q_{62}(t) © B_2(t),$$

$$B_7(t) = V_7(t) + q_{73}(t) © B_3(t),$$

(6)

After using Laplace-stieltjes transforms for equations (6) and solving for $\tilde{B}_0(s)$. In steady state the total fraction of time which the system is under repair is given by:

$$B_0(s) = \lim_{s \to 0} s\tilde{B}_0(s) = \frac{N_3(0)}{D_2'(0)}, \quad (7)$$

where,

$$N_3(0) = \mu_2\Big[\big(1 - P_{31}P_{13} - P_{15}P_{51} - P_{31}(P_{17} + P_{15}\left(P_{53}^{11} + P_{57}^{11}\right)\big)$$
$$\left(P_{02} + P_{06} + P_{04}\left(P_{42}^{10} + P_{46}^{10}\right)\right)\Big] + \mu_3\left[\left(P_{23}^{8} + P_{27}^{8}\right)\left(1 - P_{15}P_{51}\right)\right.$$

$$\left(P_{02}+P_{06}+P_{04}\left(P_{42}^{10}+P_{46}^{10}\right)\right)\Big]+\mu_4 P_{04}\Big[\left(1-P_{31}P_{13}-P_{15}P_{51}-\right.$$
$$\left.P_{31}(P_{17}+P_{15}\left(P_{53}^{11}+P_{57}^{11}\right)\right)-\left(1-P_{15}P_{51}\right)\left(P_{32}^{9}+P_{36}^{9}\right)\left(P_{23}^{8}+P_{27}^{8}\right)\Big]+$$
$$P_{15}\mu_5\left[P_{31}\left(P_{23}^{8}+P_{27}^{8}\right)\left(P_{02}+P_{06}+P_{04}\left(P_{42}^{10}+P_{46}^{10}\right)\right)\right]+$$
$$\mu_6\left(P_{06}+P_{04}P_{46}^{10}\right)\left(1-P_{13}P_{31}-P_{15}P_{51}-P_{31}\left(P_{17}+P_{15}\left(P_{53}^{11}+P_{57}^{11}\right)\right)\right.$$
$$\left.-P_{32}^{9}\left(1-P_{15}P_{51}\right)\left(P_{23}^{8}+P_{27}^{8}\right)\right)+P_{36}^{9}\left(P_{02}+P_{04}P_{42}^{10}\right)\left(1-P_{15}P_{51}\right)\left(P_{23}^{8}+P_{27}^{8}\right)\Big]$$
$$+\mu_7\Big[\left(P_{02}+P_{06}+P_{04}\left(P_{42}^{10}+P_{46}^{10}\right)\right)\left\{P_{31}P_{23}^{8}\left(P_{17}+P_{15}P_{57}^{11}\right)+P_{27}^{8}\right.$$
$$\left.\left(1-P_{31}P_{13}-P_{15}P_{51}-P_{31}P_{15}P_{53}^{11}\right)\right\}\Big].$$

PROFIT ANALYSIS

At steady state the net expected gain per unit of time is:

$$G=\lim_{t\to\infty}G(t)/t=C_1A_0-C_2B_0, \tag{8}$$

where,

C_1: is revenue per unit uptime by the system.

C_2: is per unit repair cost.

GRAPHICAL REPRESENTAION OF THE SYSTEM

Setting $\alpha_2=0.2$, $\gamma_1=0.01$, $\gamma_2=0.03$, $\theta_1=0.03$, $\theta_2=0.02$, $\beta_1=0.04$, $\beta_2=0.01$, $\delta=0.04$, $p=0.5$, $q=0.5$ and $C_1=300$, $C_2=10$ .in equations (5),(6),(7)and(8)we get Table 1.

Table 1

α_1	MTSF	Availability	Busy period	Profit
0.02	71.667	0.96534	0.9189	280.411
0.03	53.892	0.7938	0.8060	230.093
0.04	44.819	0.70666	0.7558	204.437
0.05	39.145	0.6528	0.7286	188.558
0.06	35.199	0.6159	0.7124	177.65
0.07	32.273	0.5889	0.7021	169.649
0.08	30.004	0.5682	0.6953	163.506
0.09	28.188	0.5518	0.6908	158.63
0.1	26.698	0.5384	0.6876	154.658
0.11	25.453	0.5274	0.6855	151.357
0.12	24.395	0.5180	0.6840	148.566
0.13	23.484	0.5100	0.6830	146.175
0.14	22.692	0.5030	0.6824	144.102
0.15	21.997	0.4970	0.6821	142.287

Table 1: Relation Between α_1 and the MTSF, Availability, Busy period and Profit

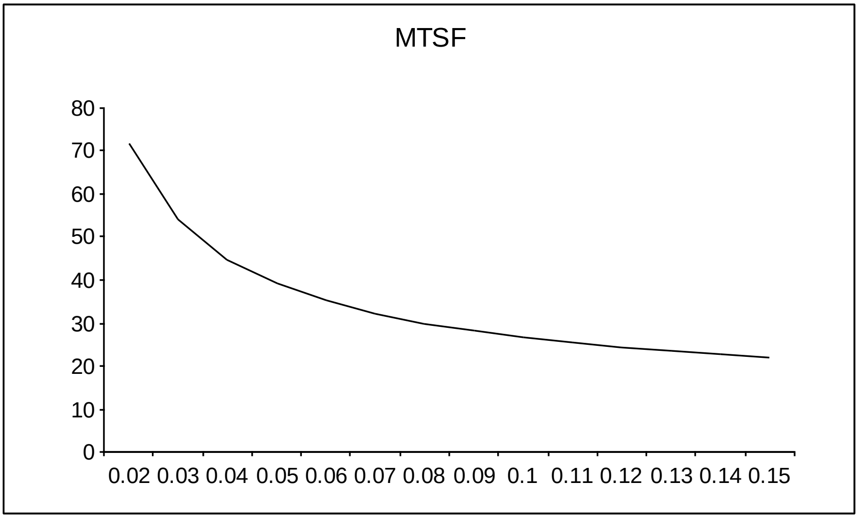


Figure 2: represent relation between α_1 and MTSF.

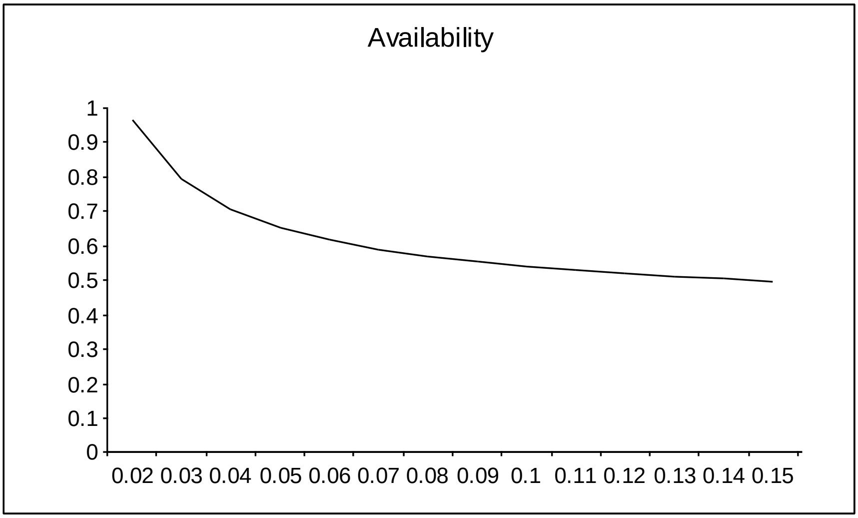


Figure 3: represent relation between α_1 and Availability.

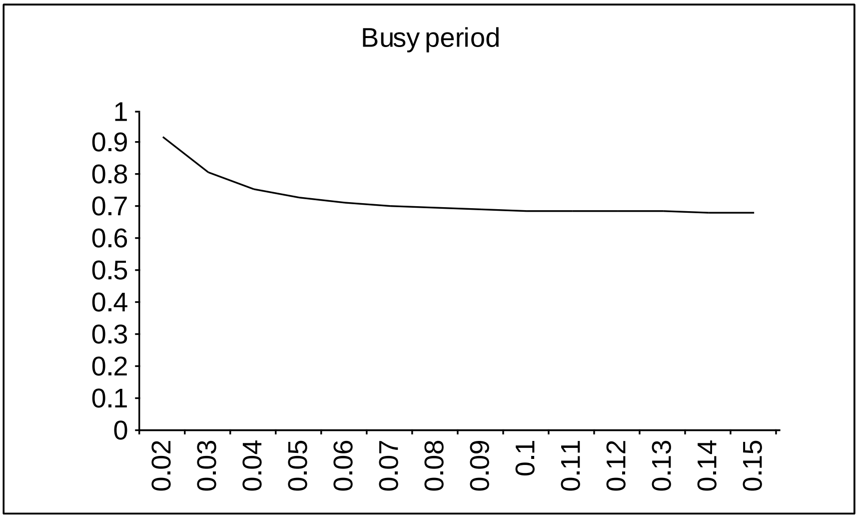


Figure 4: represent relation between α_1 and busy period.

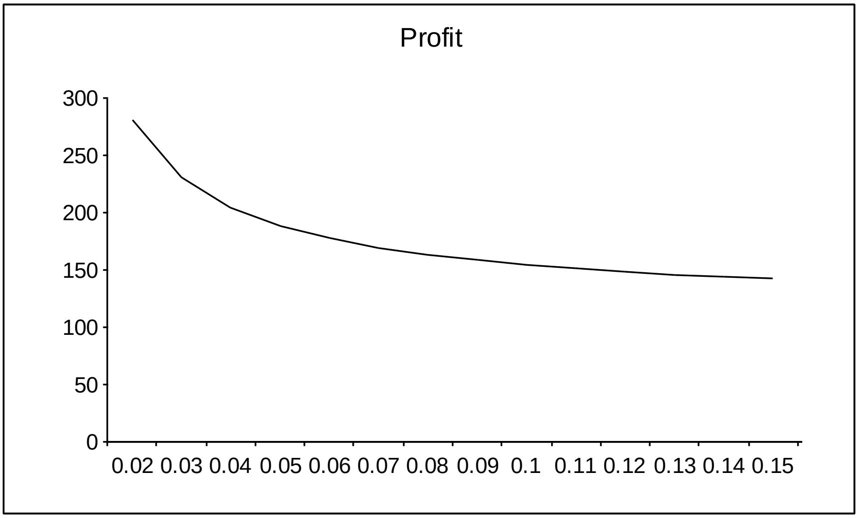


Figure 5: represent relation between α_1 and Profit

CONCLUSION

For a more concrete study of mean time to system failure, availability ,busy period and profit .we plot these characteristics w.r.t. α_1(constant failure rate of the operative from the normal mode to the total failure mode), $\alpha_2 = 0.2$, $\gamma_1 = 0.01$, $\gamma_2 = 0.03$, $\theta_1 = 0.03$, $\theta_2 = 0.02$, $\beta_1 = 0.04$,

$\beta_2 = 0.01$, $\delta = 0.04$, $p = 0.5$, $q = 0.5$. The curves and table so obtained are shown in figs. (2-5).and table 1.From these figures and table it is observed that.

1. The increase of failure rate α_1 at constant the MTSF of the system is decrease.
2. The increase of failure rate α_1 at constant the steady-state availability of the system is decrease.

3. The increase of failure rate α_1 at constant the steady-state busy period of the system is decrease.
4. The increase of failure rate α_1 at constant the profit of the system is decrease.

REFERENCES

[1] **GOEL, L, R., and GUPTA, R**., "Analysis of a two unit standby system with three modes and imperfect switching ", Microelectron. Reliab. , Vol. 24, No. 3, pp. 425-429, (1984).

[2] **GOPALAN, M.N.,** "Availability and reliability of one-server two-unit system with imperfect switchover", IEEE Trans. Reliab. R-24, 218, (1975).

[3] **MOKADDIS, G.S., ELIAS, S.S. And LABIB, S.W.,** "Probabilistic analysis of a two-unit system with a warm standby subject to preventive maintenance and single service facility ", Micro electron. Reliab, Vol.27, No.2,pp.327-343, (1987).

[4] **MOKADDIS, G.S., ELIAS, S.S. And EL-SAID KH.M** "Two models for dissimilar unit cold standby redundant system with partial failure and two types of repairs", Micro electron. Reliab, Vol.30, No.3, pp.431-451, (1990).

[5] **MOKADDIS, G. S., M. SALAH EL-SHERBENY and ENTESAR AL-ESAYEH** " Human error and partial hardware failure modeling

of parallel and standby redundant system ", Jour. of Inst. of Maths.& Computer Sciences(Comp. Sc.Ser.),Vol.19,No.2,pp.141-152, (2008).

[6] **RAJIV GARG,** "Busy period analysis of a two-unit warm standby system with imperfect switch", Micro electron. Reliab, Vol.25, No.4, pp.675-679,(1985).