ANALYSIS OF ORTHOTROPIC RECTANGULAR PLATES

Ziad H. Abo Mustafa
Dept. of Civil Engineering
Faculty of Engineering
Al Mergb University

Dr. Ali. S. Zregh
Dept. of Civil Engineering
Faculty of Engineering
Tripoli University

Abstract:

Plates are structural members that can be a slab, raft foundation in building. They can be also part of vessels, airplanes or parts of some machines. Plates can be formed by isotropic materials or orthotropic materials or some times laminated composite materials. This paper is devoted to the analysis of orthotropic rectangular plates of discontinuous line boundaries using finite difference method as a tool for such analysis. General forth order differential equation for orthotropic plate equation and its corresponding coefficient patterns of unequal spacing mesh were developed for the plate. A computer program was prepared and used to analyze the plates subjected to different loading with the variation of different Poisson's ratio and different modulus of elasticity in x&y

directions. It was found that the value of deflection decreases when the Poisson's ratio increases. It was concluded that the Poisson's ratio has a significant effect on bending moment of the plate. More sets of conclusions and recommendations will be listed by illustrating them in tabulated format and / or graphical diagrams that can be helpful to the engineers that need to benefit from these contribution.

Keyword: Raft Foundation, Finite Difference, Rectangular Plates, Isotropic, Orthotropic, Line Boundary Condition.

Introduction:

Since Navier in 1820 solved Lagrange equation of the isotropic plate bending problems employed the double Fourier series for a load and deflection relationships. Later an important approach was developed by M. Levy in 1900. When his solution is compared with the Navier's method instead of a double series he dealt with a single series. Other important solution for the problems of plate bending is the energy method by Ritz.

Many references were assigned as text books for plate analysis such as Timoshenko[3], but with the high speed of personal computers, orthotropic plates began to grasp engineering attention looking for exact analysis of plates to reach better economy of material usage and high accuracy.

In literature review solution of an orthotropic simply supported rectangular plate with uniform distributed load was illustrated in details[2,4].

This paper is devoted to the analysis of discontinuous line boundaries for rectangular orthotropic plates using finite difference method as analysis tool. General forth order differential equation for orthotropic plate equation and its corresponding coefficient patterns of unequal spacing mesh were developed. A computer program was prepared and used to solve plates with different loadings with variations in Poisson's ratio and different modulus of elasticity in x&y directions[5].

Finite Difference Method:

The method of finite differences is a tool to write sets of equations for a specific purpose. In our case solution of Lagrange equation is modified for an orthotropic plate as shown in Eq. (1).

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q \qquad (1)$$

$$H = D_{xy} + 2G_{xy} \qquad (2)$$

Where

D_x, D_y	are flexural rigidities of an orthotropic plate.
G_{xy}	torsional rigidity of an orthotropic plate.
q,	intensity of load per unit area.

This method is applied here to replace the plate governing differential equation and the expressions defining the boundary conditions with equivalent difference equations. The solution of the bending problem thus reduces to the simultaneous solution of a set of algebraic equations written for every nodal point within the plate.

Coefficient Patterns of Rectangular Mesh for Orthotropic Plate:

The coefficient patterns of rectangular mesh are diagrammatically shown in Fig.1.

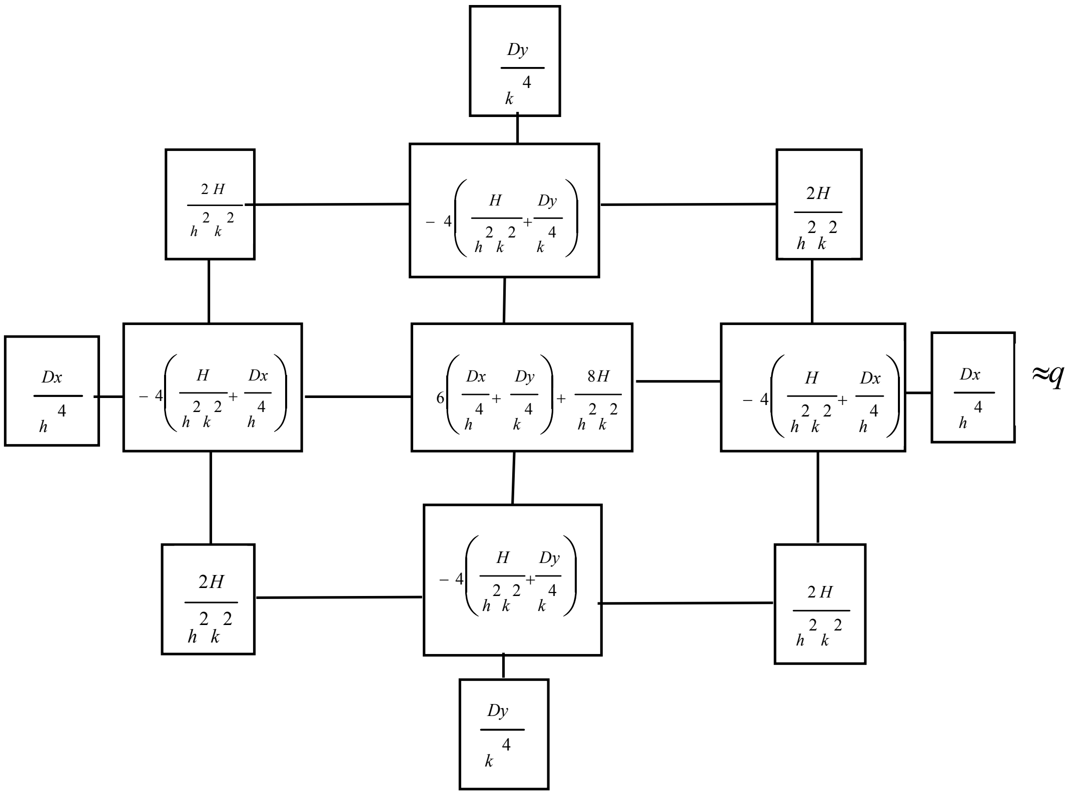

Fig. 1. Coefficient patterns of rectangular mesh for orthotropic plate [1].

Discontinuous Line Boundary Edges:

A discontinuous line boundary edge at one transient boundary point is shown in Fig. 2. The transient point on the edge of the plate which lies between the simply supported line and the fixed line is considered as sudden change point.

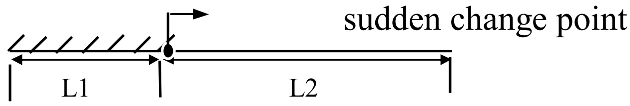

Fig. 2. Transient boundary point

Typical Problem:

An orthotropic rectangular plate with discontinuous line edge at all sides was analyzed. The edges that is subjected to different types of loads as shown in Fig. 3. The plate is subjected to four cases of the loadings, a uniform load, a triangular prism load, and triangular load of q per unit area. Fig. 3 shows the four cases of loadings. A computer program named Ortho. was developed for computing the deflection w, bending moments M_x& M_y, and twisting moment M_{xy} at a grid of points with an even number of intervals along each side of the rectangular plate. The plate dimensions considered are 6×4 meters, plate thickness equal to 0.12 meter, modulus of elasticity in x&y directions are 18 GPa and 12 GPa respectively, and the Poisson's ratios in x&y directions are 0.3 and 0.2 respectively are considered. The results were tabulated and graphs are plotted, and finally the results and graphs were compared and discussed.

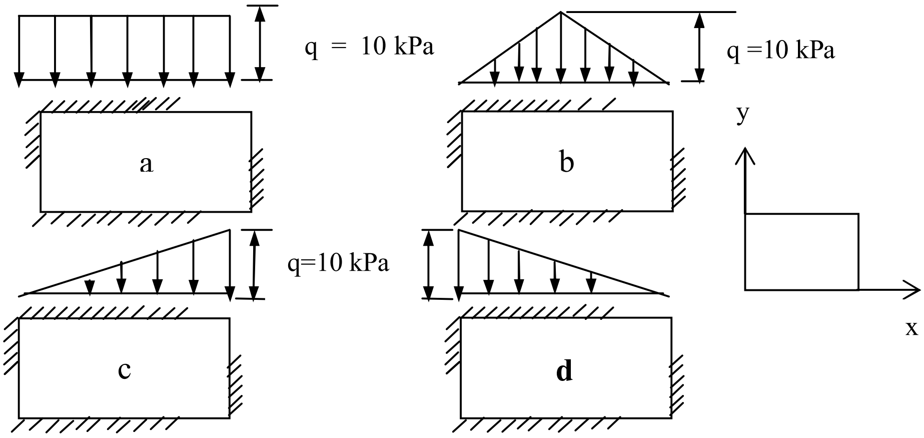

Fig. 3. Cases of loadings of a selected orthotropic plate (2a/3 edge fixed + a/3 edge simply supported)

Table (1). Illustrates numerical values of deflection, bending moments, and twisting moments at the plate center. Number of grid spacing was taken as 18 × 18. In these results the maximum intensity of load is equal to 10 kPa.

Table (1). Numerical values of deflection, bending moments, and twisting moment at the plate center

Plate type &Plate loading	w center M	M_x center N.m	M_y center N.m	M_{xy} center N.m
(a)	0.30889E -02	0.36232E+04	0.56175E+04	- 0.13642E03
(b)	0.21604E-02	0.32920E+04	0.40223E+04	- 0.68328E02
(c)	0.15754E-02	0.17981E+04	0.28456E+04	- 0.77554E02
(d)	0.15135E-02	0.18251E+04	0.27719E+04	- 0.58869E02

When a general load of intensity is considered as q, then deflection and bending moments at plate center can be expressed in terms of corresponding factors K,X, and Y as shown in the following equations.

$w = K\,qb^4$ (3)

$M_x = X\,qb^2$ (4)

$M_y = Y\,qb^2$ (5)

where

w	deflection at center of the plate.
q	load intensity per unit area.
b	width of the plate.
M_x and M_y	are bending moments in x & y directions.

Fig. 4 shows the relation between the ratio of plate dimensions against deflection factor for each load case while Figure 5 shows the relation between the same plate dimensions ratio and the positive bending moment in x & y directions.

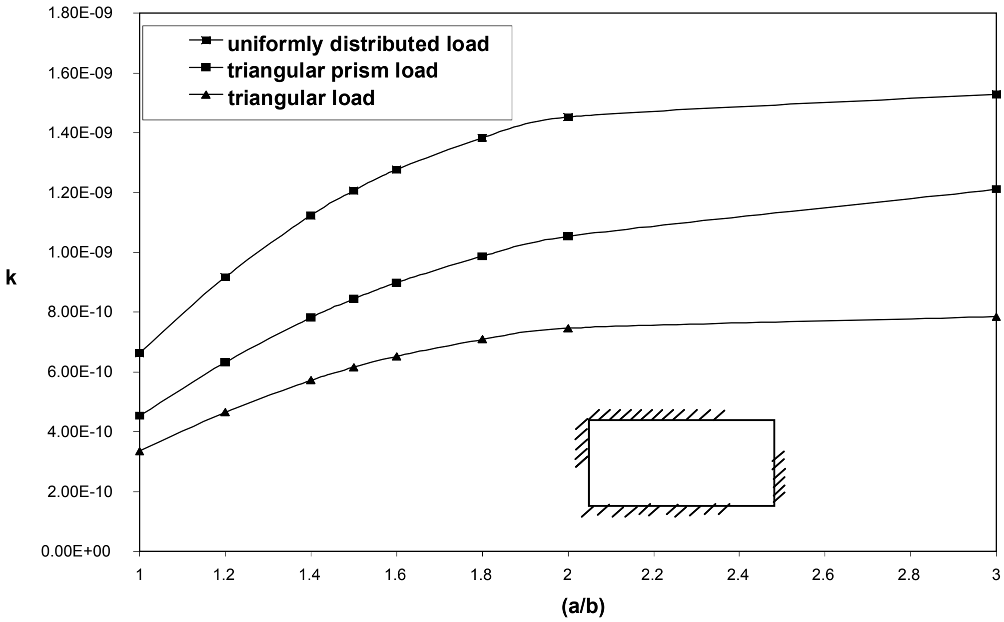

Fig. 4. Ratio of plate dimensions Vs. deflection factors at center of orthotropic plate for different types of load.

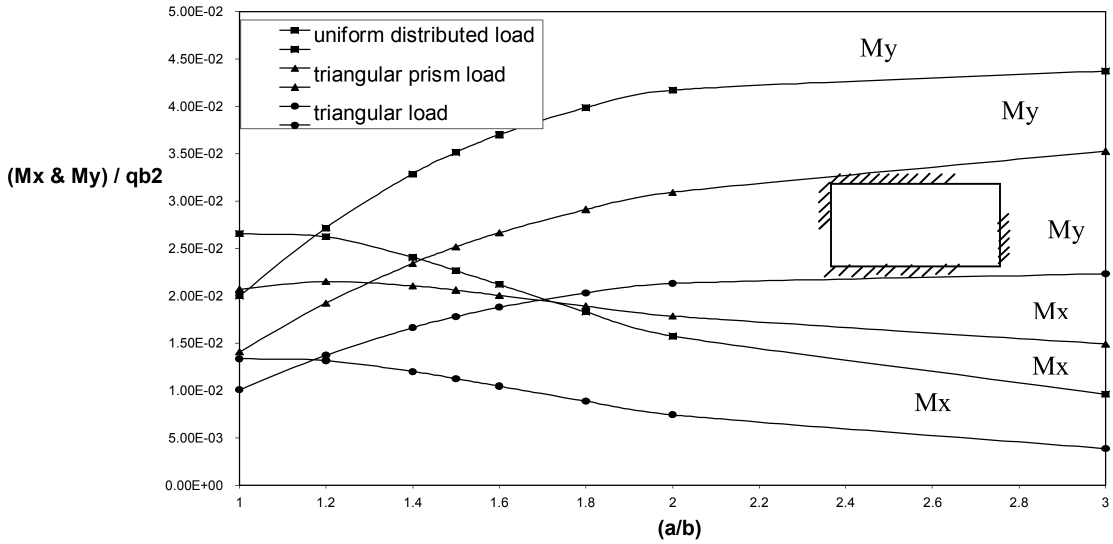

Fig. 5. Ratio of plate dimensions Vs. positive bending moments in x&y direction factors at center of orthotropic plate for different types of load.

It can be noticed from Fig. 4 that the values of deflections are increased significantly when the ratio of plate dimensions a/b varies from two way into one way plate and afterward the increase is less, on the other hand Fig. 5 indicates that the values of positive bending moments in x-direction is decreasing approximately to the half value when the ratio of plate dimensions a / b varies from one to three, while the values of positive bending moment in y-direction increased approximately twice and half when the ratio of plate dimensions a/b varies from one to three. Also it can be noticed that the values of positive bending moments in x & y direction are the same when the ratio of plate dimensions is approximately 1.2. It is noticed also that these curves indicate the points where the value of positive bending moments in x & y directions are reversed and, are located in the range of plate dimension ratios a / b of 1.2 and 1.3.

The curves in Fig. 6 and Fig. 7 show that the effect of increase in Poisson's ratio on deflection and bending moments in x & y directions of the plate, the first half of one edge is fixed and the second half is simple the other three edges are kept as simply supported.

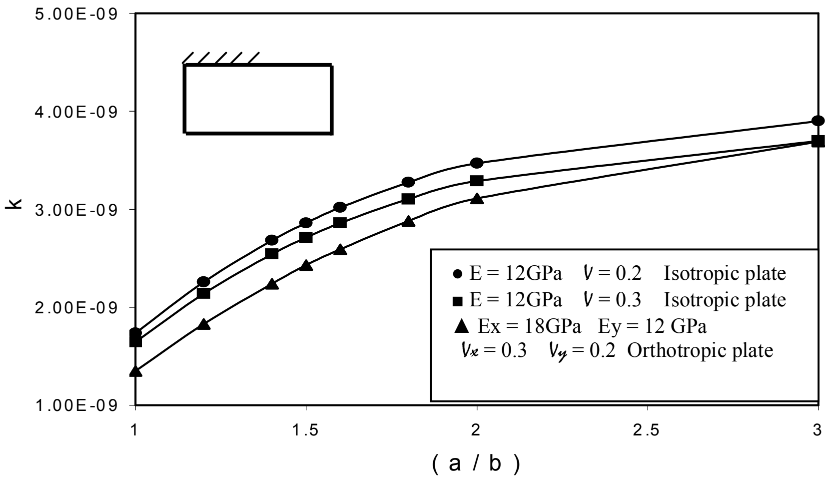

Fig. 6. Ratio of plate dimensions Vs. deflection factors at center of isotropic and orthotropic plate under uniform distributed load of 10 kPa.

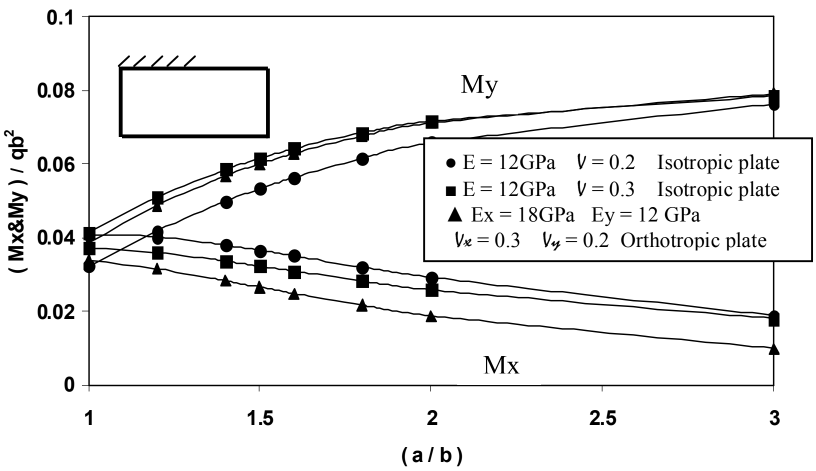

Fig. 7. Ratio of plate dimensions Vs. bending moment factors in x & y direction at center of isotropic and orthotropic plate under uniform distributed load of 10 kPa.

Fig. 6 shows that If the value of Poisson's ratio increases this will decrease the value of deflection. and Fig. 7 Illustrates that the effect of increase in Poisson's ratio has more effect on the bending moments in the long direction of the plate (bending moments in x-direction).

Line Edge Boundary Effect of Orthotropic Plates:

Fig. 8 Illustrates the eight different line boundary conditions of an orthotropic plate subjected to a 10 KPa uniform load. The purpose is to study the effect of increase in the length of fixation on the capacity of the plate, and the effect on stresses in x&y directions on the edge line of the plate, and finally to compare the positive bending moment at a transverse line crossing the center of the two opposite boundary conditions.

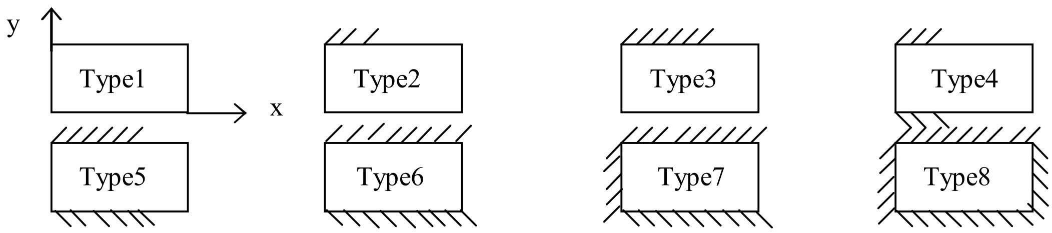

Fig. 8.Orthotropic plates with different line boundary conditions subjected to equal uniform loading.

Table (2) gives the magnitude of maximum stress and the capacity value for the eight different types of orthotropic plates subjected to a uniform distributed load of 10 kPa. It is noticed that the capacity of the section is increased approximately 106 % when the plate varies from type one (simply supported at all sides) to type eight (fixed at all sides). With a maximum stresses allowed in concrete to be equal to $f_c = 0.4 f'_c$ in which f'_c= 30 MPa. The significant increase in the capacity is noticed between type 4 and type 5 plates which indicates that the increase in 2a/3 of edge fixation along the line boundary can increase the capacity of the plate a significant amount.

Table (2) The variation in the maximum stresses and capacity for different types of boundary conditions (10 kPa uniform load).

Type number	Maximum stresses MPa	The percentages of decrease in stresses	Capacity kPa	The percentages of increase in capacity
1	4.4490	0	26.97	0
2	3.9542	11.12	30.35	12.53
3	3.5454	20.31	33.85	25.51
4	3.4835	21.70	34.45	27.73
5	2.6563	40.29	45.18	67.52
6	2.5273	43.19	47.48	76.05
7	2.3698	46.73	50.64	87.76
8	2.1642	51.36	55.45	105.6

Fig. 9 shows three types of orthotropic plates with different boundary conditions: (type 1) represents two edges partially fixed with length a/3 the remaining length is simply supported, the other two edges are fully simply supported, (type 2) represents two edges partially fixed with length 2a/3 the remaining length is simply supported, the other two edges are fully simply supported, and (type 3) represents two edges totally fixed and the other two edges are totally simply supported.

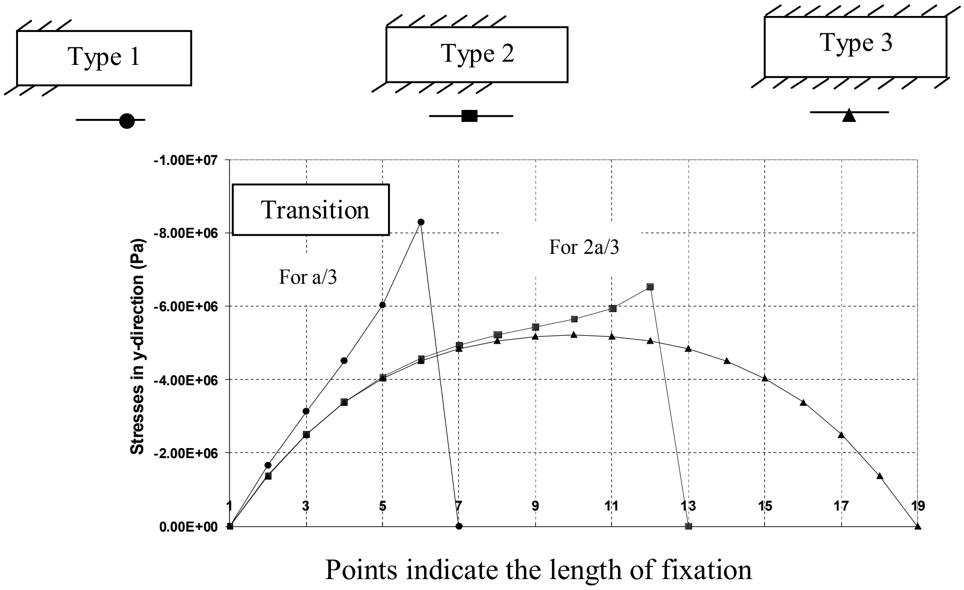

Fig. 9. Variation in stresses with increase in the length of fixation along the edge of orthotropic plates grid spacing (18 x 18).

This figure indicates that the maximum values of stresses in y direction are higher when the length of fixation is a/3. These values are decreased for the length of fixation 2a/3, and also for totally fixed edges.

Conclusions:

This paper was devoted to the analysis of orthotropic plates with different boundary conditions and concentrated on the effect of different line boundary conditions for different types of loadings.

Finite difference method was used as a tool for the analysis. Variation of Poisson's ratio and modulus of elasticity in x&y directions were considered.

From this paper which depends on solving of numerical and non-dimensional problem of orthotropic plates major findings are listed as follows:

1. Finite difference method is a good tool to solve orthotropic plates.
2. If the value of Poisson's ratio increases this will decrease the value of deflection.
3. The effect of increase in Poisson's ratio has more effect on the bending moments in the long direction of the plate (bending moments in x-direction).
4. The transient point on the edge of the plate which lies between the simply supported line and the fixed line is considered as a sudden change point .
5. The increase of fixation of boundary edges for a plate will decrease significantly the values of deflections and the positive bending moments.
6. The capacity of the plate section is increased significantly when more than 2/3 of parallel sides are fixed. This increase reaches approximately 106% when the plate varies from (simply supported at all sides) to (fixed at all sides).
7. If the length of fixation varies from a/3 to 2a/3 to totally fixed edge the maximum value of stresses along this edge of the plate occurs when the length of fixation is a/3. These values are decreased for the length of fixation 2a / 3 and continue to decrease for totally fixed edges. This means that the stresses along the line boundary decrease with the increase of length fixation.

References:

[1] Abu Mustafa, Z.H., Analysis of orthotropic plates using finite difference method, M.Sc. Thesis, El-Fateh university civil engineering department, Spring 2001

[2] Ugral, A. C., Stresses in plates and shells, Mc Graw-Hill, 1981.

[3] Timoshenko, S. P. and Woinowsky-Krieger, S., Theory of plates and shells, Second Edition, McGraw-Hill, 1981.

[4] McFarland, D., Smith, B. L. and Bernhart, W. D., Analysis of plates, Spartan Books, 1972

[5] Zregh, A. S., Tawil, M. M., and Abu Mustafa, Z. H., "Analysis of rectangular orthotropic plates with variable line boundary conditions", 6 th International Conference on Concrete Technology for Developing Countries, Vol. 3, Oct. 2002, Amman-Jordan, pp.871-880.

[6] Abu Mustafa, Z.H., Zregh, A.S., Tawil, M. M., " Analysis of Orthotropic Plates with Discontinuous Line Boundaries.", 9th Arab Structural Engineering Conf. Vol.1, Nov. 29-Dec 1.2003 Abu Dhabi, UAE, pp535-542.