Preroughness And Preexactness in Topological Spaces

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Abstract:

Generalization of rough set model is an important aspect of rough set theory research. In this paper, we use atopological concepts to introduce a generalization of Pawlak approximation space. Concepts of definability for subsets in topological approximation spaces are introduced.

Several types of approximations which called pre approximations are mathematical tools to modify the approximations. In this paper we

introduce the pre exactness and pre roughness by applying the concepts of pre open sets to make more accuracy for definability of sets, and we present new types of rough definability and rough undefinability based on these approximations.

Keywords: Approximation space; Rough definability and undefinability, Prerough and Preexact sets.

Introduction:

Rough set theory, introduced by Pawlak in 1982 [14], is a mathematical tool that supports also the uncertainty reasoning but qualitatively. Rough set theory has a wide variety of applications. It can be used for information preserving data reduction, representation of uncertain or imprecise knowledge, concept classification, machine learning, knowledge discovery, data mining [20] economics [8], medical diagnosis [13], and others [21]. A basic notion of rough set theory is the lower and upper approximation, or approximation operators [14, 15, 23].

This theory can be developed in at least two different manners, the constructive and algebraic methods [24]. The constructive methods [16, 14] define rough set approximation operators using equivalence relations or their induced partitions and subsystems; the algebraic methods treat approximation operators as abstract operators. There are several definitions of constructive methods, commonly known as the element based, granule based [18, 26], and subsystem based definitions [24,27]. Each of them offers a unique interpretation of the theory.

Rough set theory in topological spaces is an important type of generalized rough set models. This model open the way for applying rich notions and results in the theory of topological spaces in the context of generalized rough set models.

In this paper, we introduce new types of rough definability and undefinability, based on the notions of pre lower and pre upper approximations and study the relations between them. We obtain eight kinds of rough definability and undefinability instead of four in case of Pawlak approximations.

1- Prerough and preexact sets:

The present section is devoted to introduce the pre exactness and preroughness by applying the concepts of pre open sets to make more accuracy for definability of sets. Let X be a subset, then X is exact if $BN(X) = \Phi$, otherwise X is rough [4]. We shell express pre rough set properties in terms of topological concepts. X is preexact (briefly, P-exact) set if $BN_P(X) = \Phi$, otherwise X is pre rough (briefly, P-rough). It is clear that X is P-exact iff $\overline{P}(X) = \underline{P}(X)$. The pawlak space subset X has two possibilities, rough or exact. The following definitions introduce new types of definability for a subset $X \subseteq U$ in general topological space (U, T).

Definition 1.1

X is said to be preexact if $\underline{P}(X) = \overline{P}(X)$, otherwise X is said to be prerough.

Definition 1.2

Let $K = (U, \mathbb{R})$ be a general knowledge base and $X \subseteq U$, $R \in \mathbb{R}$, then

(1) X is $R - definable$ iff $\underline{R}(X) = \overline{R}(X)$, otherwise X is non definable

or rough.

(2) X is pre $R - definable$ iff $\underline{P}(X) = \overline{P}(X)$, otherwise X is non pre definable or pre rough.

Proposition 1.1

Let $(U, \mathbb{R})$ be a general approximation space and $X \subseteq U$. If X is exact, then X is pre-exact.

Proof

Let X be an exact set, then $\overline{R}(X) = X = \underline{R}(X)$. Now

$\overline{R}(X) = \bigcap\{F \subseteq U : X \subseteq F \;\&\; F \in T^C\} \supseteq \bigcap\{F \subseteq U : X \subseteq F \;\&\; F \in PC(X)\}$,

since $T^C \subseteq PC(X) = \overline{P}(X)$.Also

$\underline{R}(X) = \bigcup\{G \subseteq U : G \subseteq X \;\&\; G \in T\} \subseteq \bigcup\{G \subseteq U : G \subseteq X \;\&\; G \in PO(X)\}$,

since $\subseteq PO(X) = \underline{P}(X)$.

There for, $\underline{R}(X) \subseteq \underline{P}(X) \subseteq X \subseteq \overline{P}(X) \subseteq \overline{R}(X)$. Since X is exact we get $\overline{P}(X) = X = \underline{P}(X)$, hence X is P-exact.

The converse of the above proposition is not true in general as the following example illustrates.

Example 1.2

Let $U = \{a, b, c, d\}$,$R = \{(a, a), (b, b), (b, c), (b, d)\}$,

$S = \{\{a\}, \{b, c, d\}\}$, $\beta = \{U, \Phi, \{a\}, \{b, c, d\}\}$,

$T = \{U, \Phi, \{a\}, \{b, c, d\}\}$, $T^C(X) = \{U, \Phi, \{b, c, d\}, \{a\}\}$ &

if $X = \{a, b\} \Rightarrow \underline{R}(X) = \{a\}$, $\overline{R}(X) = U$, that is X is a rough set. But $\underline{P}(X) = X \cap \underline{R}\left(\overline{R}(X)\right) = X$, $\overline{P}(X) = X \cup \overline{R}\left(\underline{R}(X)\right) = X$, that is X is preexact set.

Proposition 1.2

Let (U, R) be a general knowledge base and$X \subseteq U$. If X is R-exact, then X is pre R-exact.

Proof

If $\underline{R}(X) = \overline{R}(X)$, then $\underline{P}(X) = X \cap \underline{R}\left(\overline{R}(X)\right) = X \cap \underline{R}\left(\underline{R}(X)\right)$

$= X \cap \underline{R}(X) = X$,

$\overline{P}(X) = X \cup \overline{R}\left(\underline{R}(X)\right) = X \cup \overline{R}\left(\overline{R}(X)\right) = X \cup \overline{R}(X) = X$.

i.e. $\underline{P}(X) = \overline{P}(X)$.

The following example shows that the converse of the previous proposition is not in general true.

Example 1.3

Let $U = \{a, b, c, d\}$ and R be a general relation on U such that $R = \{(a, a), (a, b), (d, d)\}$, has the following class, $\frac{U}{R} = S = \{\{a, b\}, \{d\}\}$ & $\beta = \{U, \Phi, \{a, b\}, \{d\}\}$, $T = \{U, \Phi, \{a, b\}, \{d\}, \{a, b, d\}\}$,

$T^C = \{U, \Phi, \{c,d\}, \{a,b,c\}, \{c\}\}$, if $X = \{a,c,d\}$, then $\overline{R}(X) = U \Longrightarrow \underline{P}(X) = \{a,c,d\}$. Also,

$\underline{R}(X) = \{d\}, \overline{R}\left(\underline{R}(X)\right) = \{c,d\} \Longrightarrow \overline{P}(X) = \{a,c,d\}$ i.e. $\underline{P}(X) = \overline{P}(X)$, but $\underline{R}(X) \neq \overline{R}(X)$.

Proposition 1.3

Let $K = (U,\mathbb{R})$ be a general knowledge base and $X, Y \subseteq U$ and

$R \in \mathbb{R}$, If X and Y are $R - definable$ then:

(1) $\underline{R}(X \cup Y) = \underline{R}(X) \cup \underline{R}(Y)$.

(2) $\overline{R}(X \cap Y) = \overline{R}(X) \cap \overline{R}(Y)$.

(3) $\underline{P}(X \cap Y) = \underline{P}(X) \cap \underline{P}(Y)$.

(4) $\underline{P}(X \cup Y) = \underline{P}(X) \cup \underline{P}(Y)$.

(5) $\overline{P}(X \cup Y) = \overline{P}(X) \cup \overline{P}(Y)$.

(6) $\overline{P}(X \cap Y) = \overline{P}(X) \cap \overline{P}(Y)$.

Proof

(1) $\underline{R}(X \cup Y) \supseteq \underline{R}(X) \cup \underline{R}(Y)$.

On other hand $\underline{R}(X) \cup \underline{R}(Y) = \overline{R}(X) \cap \overline{R}(Y)$, (since X & Y are $R - definable$)

$$= \overline{R}(X \cup Y)$$

$\supseteq \underline{R}(X \cup Y)$. So $\underline{R}(X \cup Y) = \underline{R}(X) \cup \underline{R}(Y)$.

(2) Similar proof as (1).

(3) $\underline{P}(X \cap Y) \subseteq \underline{P}(X) \cap \underline{P}(Y)$, by (Proposition 1.4). On the other hand we have

$\underline{P}(X) \cap \underline{P}(Y) \subseteq \overline{R}(X) \cap \overline{R}(Y) = \underline{R}(X) \cap \underline{R}(Y) = \underline{R}(X \cap Y) \subseteq \underline{P}(X \cap Y)$.

(4) Similar proof as (1).

(5) $\overline{R}(X \cup Y) \supseteq \overline{R}(X) \cup \overline{R}(Y)$. On the other hand we have

$$\overline{P}(X) \cup \overline{P}(Y) \supseteq \underline{R}(X) \cup \underline{R}(Y) = \overline{R}(X) \cup \overline{R}(Y) = \overline{R}(X \cup Y)$$

$\supseteq \overline{P}(X \cup Y)$.

(6) Similar proof as (5).

The following definition interprets some kinds of definability called one sided definability.

Definition 1.3

Let $K = (U,\mathbb{R})$ be a general knowledge base, $X \subseteq U$ & $R \in \mathbb{R}$.

(1) X is totally $R - definable$ (exact) set if $\underline{R}(X) = X = \overline{R}(X)$.

(2) X is internally $R - definable$ set if $\underline{R}(X) = X, \overline{R}(X) \neq X$.

(3) X is externally $R - definable$ set if $\overline{R}(X) = X, \underline{R}(X) \neq X$.

(4) X is pre-totally $R - definable$ (pre-exact) set if $\underline{P}X = X = \overline{P}X$.

(5) X is pre-internally $R - definable$ set if $\underline{P}X = X, \overline{P}X \neq X$.

(6) X is pre-externally $R - definable$ set if $\overline{P}X = X, \underline{P}X \neq X$.

(7) X is $R-indefinable$ (rough) set if $\overline{R}X \neq X, \underline{R}X \neq X$.

(8) X is pre $R-definable$ (pre rough) set if $\overline{P}X \neq X, \underline{P}X \neq X$.

The following propositions illustrate the relationship between one and two sided definability.

Proposition 1.4

Let $K=(U,\mathbb{R})$ be a general knowledge base, $X \subseteq U$ & $R \in \mathbb{R}$.

(1) X is $R-definable$ iff X is internally $R-definable$ and externally $R-definable$.

(2) X is pre $R-definable$ iff X is pre-internally $R-definable$ and pre-externally $R-definable$.

Proof

Follows immediately by Definitions 1.2 and 1.3.

Remark 1.1

If X is internally (pre-internally) or externally (pre-externally) $R-definable$ so X is $R-definable$ (pre$-definable$) is not in general true.

The following example shows this idea.

Example 1.4

Let $U=\{a,b,c,d,e\}$ and R be a general relation on U defined as $R=\{(a,a),(a,c),(c,b),(c,c),(c,d),(e,a),(e,e)\}$,

$S = \{\{a,c\},\{b,c,d\},\{a,e\}\}$, $\beta = \{U,\Phi,\{a,c\},\{b,c,d\},\{a,e\},\{c\},\{a\}\}$,
$T = \{U,\Phi,\{a,c\},\{b,c,d\},\{a,e\},\{c\},\{a\},\{a,c,e\},\{a,b,c,d\}\}$,

$T^C = \{U,\Phi,\{b,d,e\},\{a,e\},\{b,c,d\},\{a,b,d,e\},\{b,c,d,e\},\{b,d\},\{e\}\}$.

Let $X = \{a,b,c\}$, then $\overline{R}(X) = U, \underline{R}\left(\overline{R}(X)\right) = U$ &

$\underline{P}(X) = X \cap \underline{R}\left(\overline{R}(X)\right) = X$. On other hand

$\underline{R}(X) = \{a,c\}, \overline{R}\left(\underline{R}(X)\right) = U$ & $\overline{P}X = X \cup \overline{R}\left(\underline{R}(X)\right) = U \Longrightarrow$

$\underline{P}(X) \neq \overline{P}X$.

Let $X = \{x_1,x_4,x_5\}$, then $\underline{R}(X) = \{x_1,x_5\}$, $\overline{R}\left(\underline{R}(X)\right) = \{x_1,x_5\}$ &

$\overline{P}X = X$.

On other hand $\overline{R}(X) = \{x_1,x_2,x_4,x_5\}$&$\underline{P}(X) = \{x_1,x_5\} \neq \overline{P}X$.

The following proposition illustrates the relation between one sided R- $definable$ and one sided pre R- $definability$.

Proposition 1.5

Let $K = (U,\mathbb{R})$ be a general knowledge base, $X \subseteq U$ and $R \in \mathbb{R}$.

(1) If X is internally $R - definable$, then X is pre-internally $R - definable$.

(2) If X is externally $R - definable$, then X is pre-externally $R - definable$.

Proof

(1) If $\underline{R}(X) = X$ & $\overline{R}(X) \neq X,$ since $\underline{R}(X) \subseteq \underline{P}(X) \subseteq X,$ then

$\underline{P}(X) = X$ & $\overline{P}(X) = X \cup \overline{R}\left(\underline{R}(X)\right) = X \cup \overline{R}(X) = \overline{R}(X)$, i.e.$\overline{P}(X) \neq X.$

Hence X is pre-internally $- \mathit{definable}.$

(2) If $\underline{R}(X) \neq X$ & $\overline{R}(X) = X$, since $X \subseteq \overline{P}(X) \subseteq \overline{R}(X)$, then $\overline{P}(X) = X$ & $\underline{P}(X) = X \cap \underline{R}\left(\overline{R}(X)\right) = X \cap \underline{R}(X) = \underline{R}(X)$, i.e. $\underline{P}(X) \neq X$. Hence X is pre-externally $R\text{–}\ \mathit{definable}.$

In the following example we show that the converse of the previous proposition is not in general true.

Example 1.5

Let $U = \{a, b, c, d, e\ \}$ and R be a general relation on U defined as in (Example 1.4) & if $X = \{a, b, c\}$, then $\underline{R}(X) = \{a, c\}$, $\underline{P}(X) = X$, but $\underline{R}(X) \neq X$, i.e. $\underline{P}(X) = X$ does not in general imply that $\underline{R}(X) = X$.

Also if $X = \{a, d, e\}$, then $\overline{R}(X) = \{a, b, d, e\}$, $\underline{R}\left(\overline{R}(X)\right) = \{a, e\}$ & $\overline{P}X = X$, but $\overline{R}(X) \neq X$.

i.e.$\overline{P}X = X$ dose not in general imply that $\overline{R}(X) \neq X$

The following definition introduces other types of definability based on the notions of lower and upper approximations beside prelower and preupper approximations.

Definition 1.4

Let $K = (U, \mathbb{R})$ be a general knowledge base, $X \subseteq U$ & $R \in \mathbb{R}$.

(1) X is roughly $R-$ $definable$set if $\underline{R}(X) \neq \Phi$ & $\overline{R}(X) \neq U$.

(2) X is pre roughly $R-$ $definable$set if $\underline{P}(X) \neq \Phi$ & $\overline{P}(X) \neq U$.

(3) X is internally $R - undefinable$ set if $\underline{R}(X) = \Phi$ & $\overline{R}(X) \neq U$.

(4) X is pre-internally $R - undefinable$ set if $\underline{P}(X) = \Phi$ & $\overline{P}(X) \neq U$.

(5) X is externally $R - undefinable$ set if $\underline{R}(X) \neq \Phi$ & $\overline{R}(X) = U$.

(6) X is pre-externally $R - undefinable$ if $\underline{P}X \neq \Phi$ & $\overline{P}(X) = U$.

(7) X is totally $R - undefinable$ If $\underline{R}(X) = \Phi$ & $\overline{R}X = U$.

(8) X is pre-totally $R - undefinable$ if $\underline{P}X = \Phi$ & $\overline{P}X = U$.

The following proposition indicates the relation between the previous types of undefinability.

Proposition 1.6

Let $K = (U, \mathbb{R})$ be a general knowledge base, $X \subseteq U$ and $R \in \mathbb{R}$.

(1) X is roughly $R - definable$ iff X is pre roughly $R - definable$.

(2) X is pre-totally $R - undefinable$ iif X is totally $R - undefinable$.

(3) X is pre-internally $R - undefinable$ iif X is internally

$R - undefinable$.

(4) X is pre-externally $R - undefinable$ iif X is externally

$R - undefinable$.

Proof

(1) $\Longrightarrow$ Suppose that X is roughly $R - definable$, then $\underline{R}(X) \neq \Phi$ & $\overline{R}(X) \neq U$.

Since $\underline{R}(X) \subseteq \underline{P}X \subseteq \overline{P}X \subseteq \overline{R}(X)$, then $\underline{P}X \neq \Phi$ & $\overline{P}X \neq U$, i.e. X is preroughly $R - definable$.

$\Longleftarrow$Suppose that $\underline{P}X \neq \Phi$ & $\overline{P}X \neq U$, since if $\underline{R}(X) = \Phi \Longrightarrow \overline{P}X = X \cup \overline{R}\left(\underline{R}(X)\right) \neq U \Longrightarrow X \cup \overline{R}\left(\underline{R}(X)\right) = \Phi \Longrightarrow X = \Phi$,

a contradiction so $\underline{R}(X) \neq \Phi$ & if $\overline{R}(X) = U$

$\underline{P}X = X \cap \underline{R}\left(\overline{R}(X)\right) \neq \Phi \Longrightarrow X \cap \underline{R}\left(\overline{R}(X)\right) = U$ i.e. $X \cap U = U$,

a contradiction so $\overline{R}(X) \neq U$.

(2) $\Longrightarrow$ Suppose that X is pre-totally $R - undefinable$

i.e. $\underline{P}X = \Phi$ & $\overline{P}X = U$, we show that $\underline{R}(X) = \Phi$ & $\overline{R}(X) = U$,

since $\underline{R}(X) \subseteq \underline{P}X \subseteq \overline{P}X \subseteq \overline{R}(X) \Longrightarrow \underline{R}X = \Phi$ & $\overline{R}X = U$.

$\Longleftarrow$ Suppose that X is totally$R - undefinable$ i.e.$\underline{R}(X) = \Phi$ & $\overline{R}(X) = U$, we show that $\underline{P}X = \Phi$ & $\overline{P}X = U$, if $\underline{P}X \neq \Phi \Longrightarrow X \cap \underline{R}\left(\overline{R}(X)\right) \neq \Phi \Longrightarrow X \cap \underline{R}\left(\overline{R}(X)\right) = U$, i.e. $X \cap U = U$,

a contradiction so $\underline{P}X = \Phi$ & if $\overline{P}X \neq U \Longrightarrow X \cup \overline{R}\left(\underline{R}(X)\right) \neq U \Longrightarrow$

$X \cup \overline{R}\left(\underline{R}(X)\right) = \Phi$ i.e. $X = \Phi$, a contradiction so $\overline{P}X = U$.

(3) ($\Longrightarrow$) Suppose that $\underline{P}X = \Phi$ & $\overline{P}X \neq U$, then $\underline{R}(X) \subseteq \underline{P}X = \Phi$, i.e. $\underline{R}(X) = \Phi$ & suppose that $\overline{R}(X) = U$, then

$\underline{P}X = X \cap \underline{R}\left(\overline{R}(X)\right) = X \cap U = U$, a contradictions so $\overline{R}(X) \neq U$.

($\Longleftarrow$) Suppose that $\underline{R}(X) = \Phi$ & $\overline{R}(X) \neq U$, then we show that $\underline{P}X = \Phi$ &

$\overline{P}X \neq U$, if $\underline{P}X \neq \Phi \Longrightarrow X \cap \underline{R}\left(\overline{R}(X)\right) \neq \Phi \Longrightarrow X \cap \underline{R}\left(\overline{R}(X)\right) = U$

$\Longrightarrow \overline{R}(X) = U$, a contradiction so $\underline{P}X = \Phi$, and if $\overline{P}X = U \Longrightarrow$

$X \cup \overline{R}\left(\underline{R}(X)\right) = U \Longrightarrow X = U$, a contradiction so $\overline{P}X \neq U$.

(4) ($\Longrightarrow$) Suppose that $\underline{P}X \neq \Phi$ & $\overline{P}X = U$, since $\overline{P}X \subseteq \overline{R}(X)$, then

$\overline{R}(X) = U$& if $\underline{R}(X) = \Phi \Longrightarrow \overline{P}X = X \cup \overline{R}\left(\underline{R}(X)\right) = X$,

a contradictions so $\underline{R}(X) \neq \Phi$.

($\Longleftarrow$) Suppose that $\underline{R}(X) \neq \Phi$ & $\overline{R}(X) = U$, and show that $\underline{P}X \neq \Phi$ &

$\overline{P}X = U$, since $\underline{R}(X) \subseteq \underline{P}X$, so $\underline{P}X \neq \Phi$ & if $\overline{P}X \neq U \Longrightarrow$

$X \cup \overline{R}\left(\underline{R}(X)\right) \neq U$ i.e. $X \cup \overline{R}\left(\underline{R}(X)\right) = U^C \Longrightarrow X \cup \overline{R}\left(\underline{R}(X)\right) = \Phi \Longrightarrow$

$\overline{R}\left(\underline{R}(X)\right) = \Phi$, i.e. $\underline{R}(X) = \Phi$, a contradictions so $\overline{P}X = U$.

Proposition 1.7

Let $K = (U,\mathbb{R})$ be a general knowledge base, $X \subseteq U$ and $R \in \mathbb{R}$.

(1) A set X is $R - definable$ (roughly $R - definable$, totally $R - undefible$) iff so X^C.

(2) A set X is externally (internally) $R-undefible$ if X^C is internally (externally) $R-undifible.$

Proof

(1) If X is $R-definable$ then $\underline{R}(X) \neq \Phi$ & $\overline{R}(X) \neq U,$ we show that $\underline{R}(X^C) \neq \Phi$, $\overline{R}(X^C) \neq U$

Since $\underline{R}(X) \neq \Phi \Longrightarrow \underline{R}(X) = \left(\overline{R}(X^C)\right)^C \neq \Phi \Longrightarrow \overline{R}(X^C) \neq U,$ and since $\overline{R}(X) \neq U \Longrightarrow$

$\overline{R}(X) = \left(\underline{R}(X^C)\right)^C \neq U \Longrightarrow \underline{R}(X^C) \neq \Phi$ i.e. X^C is R- $definble.$

Also if X is totally $R-undefinable$ then $\underline{R}(X) = \Phi$ & $\overline{R}(X) = U,$ we show that $\underline{R}(X^C) = \Phi,$ &$\overline{R}(X^C) = U$ since $\underline{R}(X) = \Phi \Longrightarrow$

$\left(\overline{R}(X^C)\right)^C = \Phi \Longrightarrow \overline{R}(X^C) = U.$ And since $\overline{R}(X) = U \Longrightarrow$

$\left(\underline{R}(X^C)\right)^C = U \Longrightarrow \underline{R}(X^C) = \Phi.$

(2) X is externally $R-undefinable \Leftrightarrow X^C$ is internally

$$R-undefinable \Longrightarrow \underline{R}(X) \neq \Phi$$

& $\overline{R}(X) = U \Leftrightarrow \underline{R}(X^C) = \Phi$ & $\underline{R}(X^C) \neq U \Longrightarrow$

$\underline{R}(X) \neq \Phi \Leftrightarrow \left(\overline{R}(X^C)\right)^C \neq \Phi \Leftrightarrow \overline{R}(X^C) \neq U$& $\overline{R}(X) = U \Leftrightarrow$ $\left(\underline{R}(X^C)\right)^C = \Phi \Leftrightarrow \underline{R}(X^C) = \Phi.$

Also X is internally $R - undefinable$- $\Leftrightarrow X^C$ is externally

$$R - undefinable \Longrightarrow$$

$\underline{R}(X) = \Phi \Leftrightarrow \left(\overline{R}(X^C)\right)^C = U \Leftrightarrow \overline{R}(X^C) = U$ & $\overline{R}(X) \neq U \Leftrightarrow \left(\underline{R}(X^C)\right)^C \neq \Phi \Leftrightarrow \underline{R}(X^C) \neq \Phi$.

Proposition 1.8

Let $K = (U,\mathbb{R})$ be a general knowledge base, $X \subseteq U$ and $R \in \mathbb{R}$.

(1) A set X is pre$R- definable$ (preroughly $R - definable$, pre-totally $R - undefinable$) iff so X^C.

(2) A set X is pre-externally (pre-internally) R- $undefinable$ iff X^C is pre internally (pre-externally) R- $undefinable$.

Proof

Similar proof as Proposition 1.7.

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